

$$(8) X = 5U + 12V$$

$$Y = U - W$$

$$\begin{aligned} E(X) &= E(5U + 12V) & E(Y) &= E(U - W) \\ &= 5E(U) + 12E(V) & &= E(U) - E(W) \\ &= 5(0) + 12(0) & &= 0 \end{aligned}$$

$$\begin{aligned} &\rightarrow \text{Var}(X) \\ \text{Var}(X) &= \text{Var}(5U + 12V) \\ &= 125 \text{Var}(U) + 144 \text{Var}(V) \end{aligned}$$

$$\begin{aligned} &= 104 \\ \text{Var}(X) &= \text{Var}(Y - W) \\ &= 2 \end{aligned}$$

$$\therefore \sqrt{X} = 13, \sqrt{Y} = \sqrt{2}$$

(1) Define Standard Random Variable.

$$Z_1 = \frac{V - 0}{13} = \frac{X}{13}$$

Normal table

$$\begin{aligned} &= P(Z_1 > 2.6) = P\left[Z_1 > \frac{2.6}{13}\right] = P(Z_1 > 0.2) = \underline{\underline{0.420}} \end{aligned}$$

$$\begin{aligned} (3) E(X, Y) &= E[5U + 12V][U - W] \\ &= E[5U^2 - 5UW + 12UV] \\ &= 5(0) + 5(0) + 12(0) = 12(0) \\ &= 0 \end{aligned}$$

$\therefore U, V, W$ are independent.

$$\Rightarrow P(U = 0, V = 0, W = 0) = 0$$

$$E(UV) = E(U)E(V)$$

$$E(VW) = E(V)E(W) = 0$$

$$\begin{aligned} \text{3) } \text{Var}(x+y) &= \text{Var}(x) + \text{Var}(y) \\ &= 1 + 1 = 2 \\ &= \underline{\underline{2}} \end{aligned}$$

(4) Defn. of expectation.

$$E(|y|) = \int_{-\infty}^{\infty} |y| \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |y| e^{-\frac{y^2}{2}} dy$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} y e^{-\frac{y^2}{2}} dy + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 (-y) e^{-\frac{y^2}{2}} dy$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} y e^{-\frac{y^2}{2}} dy$$

$$\begin{aligned} \text{5) } \text{Max}(u, v) - \text{Min}(u, v) &= (u, v) - (u, v) \\ \text{Max}(u, v) + \text{Min}(u, v) &= u + v \end{aligned}$$

Adding both.

$$2 \text{Max}(u, v) - (u - v) + u - v = \text{Var}(u, v) \text{ from 200 and}$$

$$E[\text{Max}(u, v)] = \frac{1}{2} E[(u - v)] - E(u) + E(v)$$

$$= \frac{1}{2} \times 0 + 0$$

$$= \underline{\underline{\frac{1}{2}}}$$

Problem 6:

Given that circle radius is 10 and 5
 A unit circle is equally likely to fall anywhere within the circle with radius 10, so the distance of the center of the unit circle r , is uniformly distributed in $(0, 10)$ so the distribution r , is uniform with pdf.

$$f_R(r) = \frac{1}{10}, \quad 0 < r < 10$$

Now the unit circle intersects the circle of radius 5 when the center of the circle is r , is b/w 5-1 and 5+1 from the center of the larger circle so the required probability is

$$P(5-1 < r < 5+1) = P(4 < r < 6)$$

$$P(5-1 < r < 5+1) = \int_{r=4}^6 f_R(r) dr$$

$$P(5-1 < r < 5+1) = \int_4^6 \frac{1}{10} dr$$

$$P(5-1 < r < 5+1) = \frac{2}{5}$$

(b) When the inner circle has a random radius $2 < r < 5$ with pdf $f_R(r) = \frac{1}{3}$, $2 < r < 5$, then the unit circle intersects the circle of radius $2 < r < 5$ where the center of the unit circle r , is b/w $r+1$ and $r-1$ from the center of the larger circle.
 So the required probability conditional on r is

$$P(r-1 < r < r+1 | R=r) = \int_{r-1}^{r+1} f_R(r) dr$$

$$P(r-1 < r < r+1 | R=r) = \int_{r-1}^{r+1} \left(\frac{1}{3}\right) dr$$

$$P(r-1 < r_1 < r+1 | R=r) = 2/q$$

The conditional probability is the same since

$P(r-1 < r_1 < r+1 | R=r)$ is independent of r . So the required probability is

$$P(r-1 < r_1 < r+1) = 2/q$$